EARTHQUAKE RISK PERCEPTION, COMMUNICATION AND MITIGATION STRATEGIES ACROSS EUROPE

Piero Farabollini, Francesca Romana Lugeri, Silvia Mugnano
Editors

Il Sileno Edizioni

OPEN ACCESS
PEER-REVIEWED SERIES
Geographies of the Anthropocene

ISSN 2611-3171

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Open Access and Peer-Reviewed series

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CONTENTS

2. A Collection of Statistical Methods for Analysis of the Disaster Damages and the Seismic Regime

Vladilen F. Pisarenko[1], *Mikhail V. Rodkin*[2]

Abstract

In this paper, we present a collection of statistical methods addressing heavy tailed distributions. The empirical distributions of damage from natural disasters, both in terms of material losses and fatalities, are often modelled by theoretical distributions with a heavy power-law tail. The distribution of earthquake energy (seismic moments) is another example of such a heavy tailed distribution. The statistical methods that we discuss here allow to perform an analysis of empirical distributions at different levels depending on the amount of available data. We perform a detailed analysis of heavy tailed distribution using the theory of extreme values, and discuss the related examples. The presented methods of analysis of heavy tailed distributions constitute a toolbox, which can be useful in a number of practical applications.

Keywords: disaster related damage; power-law distribution, heavy tailed distribution; theory of extreme values; seismic regime.

Introduction

Most of the damages produced by natural disasters, such as earthquakes or typhoons, are caused by rare and strong events, which release large amounts of energy. Many authors emphasize the universality of the power law distribution when characterizing natural hazards [Turcotte & Malamud, 2004; Newman, 2005; Sornette, 2000; Kijko, 2011; O'Brien et al., 2012; Liu et al., 2014; Kousky, 2014; Smit et al., 2017; Rougier et al.,2018]. Power law distributions are observed for damages caused both by natural and by man-

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made disasters, as well as for energy values and for seismic moments of earthquakes [Sornette, 2000; Clauset et al., 2009]. The main contribution to the damage caused by disasters with a heavy-tailed distribution is due to rare strong events. Therefore, the estimation of magnitude distribution in the uppermost range plays a key role in the problems related to natural disasters hazard assessment and mitigation. The statistical theory of extreme values provides a solid mathematical base for such estimations. In this work, we mainly focus on the problem of seismic risk assessment, since the corresponding data sets are more abundant and well suited for the demonstration of different statistical methods of heavy tailed distribution analysis.

The empirical distribution of damage produced by various types of catastrophes can often be described by the Pareto power-law:

$$F(x) = 1 - (a/x)^{\beta}, \quad x \geq a. \tag{1}$$

Such power-law distribution with the exponent $\beta \leq 1$ has an infinite mathematical expectation $E(X)$:

$$E(X) = \int_a^{\infty} x dF(x) = \beta a^{\beta} \int_a^{\infty} x^{-\beta} dx = \infty.$$

Sample estimates of parameter β in the range $\beta \leq 1$ are readily obtained from empirical data sets. Of course, all real losses and earthquake energies are finite. Thus, the true law of distribution in the range of rare strong events should deviate from the Pareto law (1). The finiteness of a random value can be modelled by a distribution with a maximum possible value [Cosentino et al. 1977; Kijko and Sellevoll, 1989; 1992; Pisarenko et al., 1996; Ward, 1997; Burroughs & Tebbens, 2001; Kijko and Singh, 2011; Vermeulen & Kijko, 2017] or by a distribution with special rapidly decaying factor in the extreme range [Kagan, 1999]. We use below the finite distributions resulting from the extreme value theory: the General Extreme Value distribution (GEV) and the Generalized Pareto Distribution (GPD) with negative form parameter (see below). They naturally appear as the limit distributions for maxima of observed random values [Gumbel, 1958].

This article presents an overview of the results published in a series of works by the authors [Pisarenko & Rodkin 2010; 2014; 2015; 2017; Pisarenko et al., 2014; 2017] and it contains as well several new results.

1. A faster-than-linear growth of cumulative damage with time and its possible incorrect interpretation.

Cumulative damage from natural disasters frequently exhibits a nonlinear (faster than linear) growth with time. This growth is observed for both the number of disasters and for the associated losses (examples are shown in Figs. 1, 2).

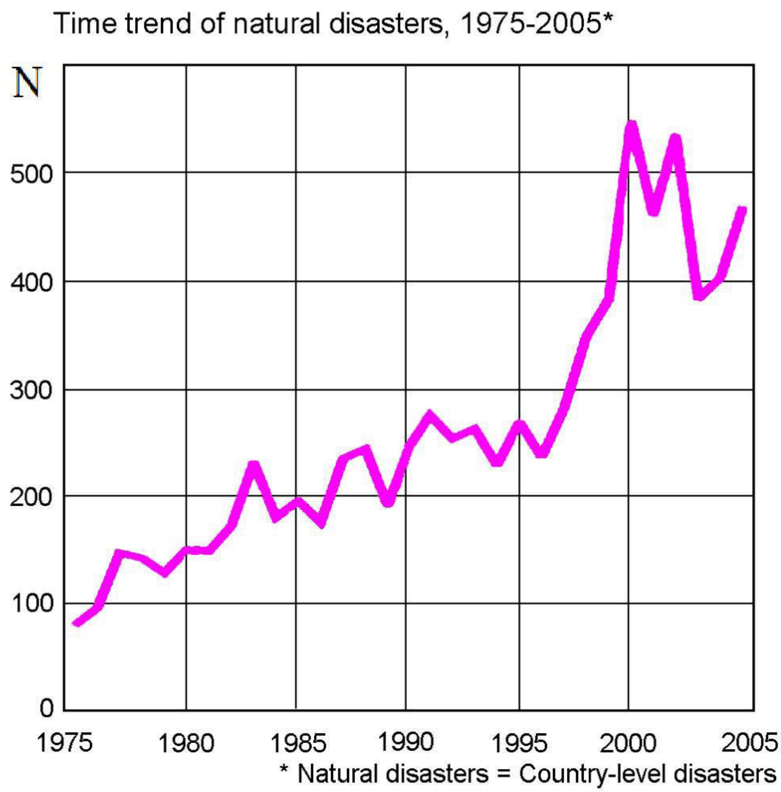

Figure 1 - *The annual number of world natural disasters as a function of time during the period 1975–2005 based on EM–DAT, [Pisarenko & Rodkin, 2010]; the definition of a country-level disaster is given in EM-DAT Glossary and means a disaster that has affected a particular country, if several countries were affected the disaster is indicated several times.*

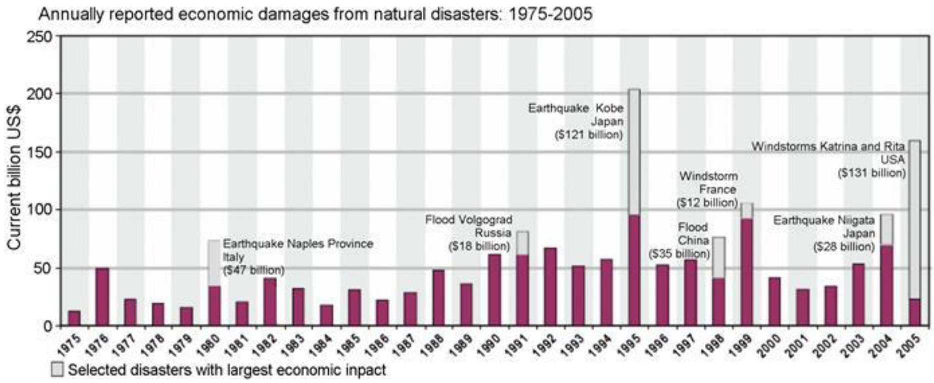

Figure 2 - *Annual losses from natural disasters (including individual most damaging events) during the period 1975–2005 based on EM–DAT, [Pisarenko & Rodkin, 2010].*

Whereas the increase of the number of documented catastrophes may be attributed, at least partly, to the development of registration systems and to a greater availability of the corresponding information, the non-linear growth of cumulative damage requires a separate explanation. Some authors attribute such faster-than-linear growth to such factors as worsening ecological conditions, urbanization, population growth, and climate change [Berz, 1992; Osipov, 1995; 2002; Seneviratne et al., 2012]. However, such growth can as well be observed in a stationary situation, provided the distribution of damage values has a heavy tail [Pisarenko, 1998; Pisarenko and Rodkin, 2010; 2014]. Let us analyze this case in more detail.

As noted above, the empirical distributions of damage produced by different types of natural disasters can be approximated by the Pareto power-law distribution (1). Examples of distribution of damage values from floods, hurricanes and earthquakes in the USA are shown on Fig. 3. The estimates of the exponent β obtained by the maximum likelihood method are less than one, thus we are dealing with heavy tailed distributions.

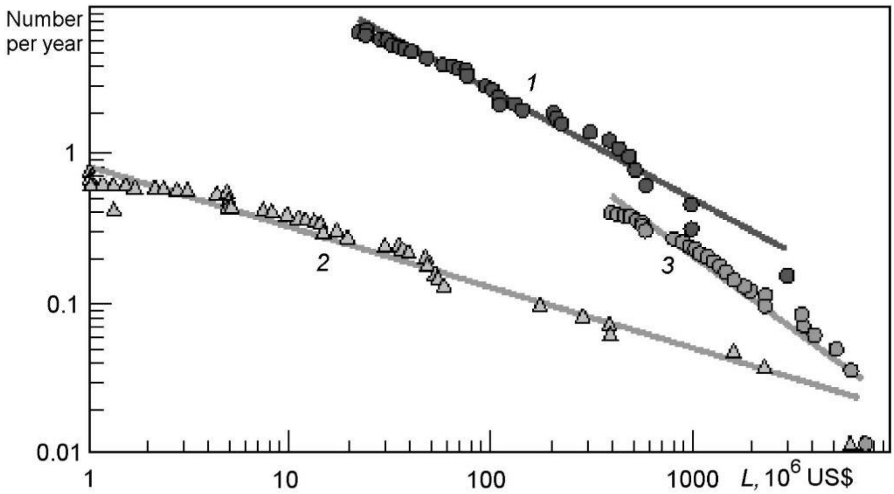

Figure 3 - *Mean numbers N of events per year with economic losses greater than L ($ USA): floods 1986-1992 (1); earthquakes 1900-1989 (2); hurricanes 1986-1989 (3).*

The fitted power-law complementary distributions 1 – F(x) are shown by lines, the values of exponents are: β_n = 0.74 (floods): β_n = 0.41 (earthquakes), and β_n = 0.98 (hurricanes) [Pisarenko & Rodkin, 2010].

The maximum likelihood estimate of parameter β equals

$$\beta_n = n/(\Sigma \ln(x_i/a)), \qquad (2)$$

where the sum is taken over all $x_i \geq a$, i=1, …n.

Now let us assume that the number of events n is a random value obeying the Poisson law

$$Pr\{n=k\}= (\lambda T)^k/ k! \quad ; k = 0,1, 2, 3,\ldots$$

with parameter λT (λ is the intensity of corresponding Poisson process, T is time of observation). Let us consider the median $\mu(T)$ of the largest event that occurred within time interval [$0\ T$]. The median $\mu(T)$ of the maximum event over time T for the Pareto law equals [Pisarenko & Rodkin, 2010]

$$\mu(T) = a\ (\lambda T/\ln(2))^{1/\beta}\ . \qquad (3)$$

Let us denote by $\Sigma(T)$ the sum of damages and by $R(\lambda T, \beta)$ the ratio $\Sigma(T)/\mu(T)$:

$$\Sigma(T) = R(\lambda T, \beta) \times \mu(T). \qquad (4)$$

It can be shown that $R(\lambda T, \beta)$ remains limited as T tends to infinity [Feller, 1966]. This property can be expressed in a different way: for distributions of type (1) with $\beta < 1$, the one strongest event is of the same order as the cumulative sum of all other events. It follows from the above formulas that the mean value $E[\Sigma(T))]$ increases linearly with T for $\beta > 1$ and proportional to $T^{1/\beta}$ for $\beta \leq 1$:

$$E[\Sigma(T))] = C(\beta,T)\cdot T^{\max(1,\, 1/\beta)} ; \quad C(\beta,T) \text{ is limited.} \tag{5}$$

As it can be seen from (3) - (5), both the single maximum event and the cumulative sum of all the events increase with time in a nonlinear manner, that is proportionally to $T^{1/\beta}$, if $\beta < 1$; and that is true even for a stationary process.

Of course, one should distinguish the statistical effect of a non-linear growth due to a heavy tail, from the effect of a real non-stationarity of the regime of disasters, caused for instance by climate change, by an increased vulnerability of the technological infrastructure to disasters, or by other factors.

As we noted above, a distribution with the exponent $\beta<1$ is not applicable to physical quantities, but for relatively small data sample sizes one can obtain sample estimates $\beta_n <1$ (see Fig.3). This may occur because small data samples with β exceeding unity are hardly distinguishable from samples with β below unity. This effect can be clearly illustrated if we consider the Gutenberg-Richter (G-R) and the truncated G-R distributions, $F_{GR}(x)$ and $F_{TGR}(x)$ respectively. $F_{TGR}(x)$ is bounded by some M_{max} value. One can achieve an arbitrarily small departure $| F_{GR}(x) - F_{TGR}(x) |$ within an arbitrarily wide interval $|x| \leq A$ by choosing a sufficiently large value of M_{max}. This means it is practically impossible to distinguish these two distributions based on small data samples. Thus, one can expect that certain statistical properties of the heavy-tailed distributions cannot be reliably estimated from small or intermediate data samples. It is very difficult to justify the choice of M_{max} for certain types of damages: the associated costs can be extremely high in today's world. But still, one may reasonably assume some kind of saturation of the growth of damages, at least for some types of damages. This saturation effect can reveal itself through a precise shape of the distribution's tail (a bend down of the tail), whereas a non-saturated sample distribution may include a huge event, such as the mega-earthquake Tohoku in Japan (2011, M = 9.08). Such events occur very rarely and can produce effects typical for the heavy-tailed distributions: in particular, they may result in a non-linear growth of total losses over a certain time interval.

2. Description of the bend down of heavy tail distributions – an approximated technique

This section provides a simple method for description of the bend down in the distribution tail, suitable for the cases of small and moderate sample sizes; see [Pisarenko and Rodkin, 2010] for details.

As discussed above, a characteristic feature of the power-law distributions with $\beta < 1$ is a nonlinear initial growth of total damage with time. A similar nonlinear growth is observed for the total released seismic energy (5):

$$E[\Sigma(T))] = C\cdot T^{1/\beta}; \qquad \beta < 1. \qquad (6)$$

However, this non-linear growth must slow down with time and eventually become linear with respect to time, which is typical of stationary processes with a finite average value:

$$E[\Sigma(T))] = C\cdot T; \qquad \beta \geq 1. \qquad (7)$$

We define the transition time T_{tp} as the time moment when the nonlinear cumulative growth becomes linear. Below we describe how one can evaluate T_{tp}. Let us define the characteristic damage value D_{tp}, with recurrence time T_{tp} corresponding to the transition point. In practice, this transition point can be estimated with a large uncertainty since the available time series of disaster related damage or those of released seismic energy are extremely variable. In order to reduce this uncertainty, we suggest the following bootstrap procedure (see for details [Pisarenko & Rodkin, 2010). We numerically simulate a number of damage curves $S(t)$, using a randomly shuffled original data sample (say, 1000 samples). Then for each time t, we take the median $MS(t)$ of the bootstrap curves $S(t)$:

$$MS(t) = median < S(t) >.$$

The median $MS(t)$ can be approximated by a simple regression relation

$$lg(MS(t)) = a_o + a_1\, lg(t) + a_2\, lg^2(t), \qquad (8)$$

where coefficients a_o, a_1, and a_2 are estimated by the standard least squares method.

For stationary time series with a finite average C we obtain

$$MS(t) = Ct, \ \ and \ \ lg(S(t)) = lg(t) + lg(C).$$

Thus,

$$d(\ lg(MS(t)) \,/\, d(\ lg(t)\ ) = 1. \tag{9}$$

So, we can take this condition for the determination of the transition point T_{tp} exploiting equations (6) - (7) as an assumed behavior of damage curves. The transition point T_{tp} is determined as the smallest t value, for which the equation (6) sill keeps satisfied. Thus,

$$a_1 + 2 \cdot a_2\ lg(T_{tp}) = 1\ ; \quad lg(T_{tp}) \ = (1 - a_1)\,/(2 \cdot a_2). \tag{10}$$

Using T_{tp} we determine the corresponding damage value D_{tp}:

$$D_{tp} = S(T_{tp}).$$

The value D_{tp} can be regarded as the characteristic damage typical of tail events. The cumulative damage $S(t)$ behaves approximatively linearly with t for $t > T_{tp}$.

The procedure described above is applicable not only to the damage data, but also to the disaster related death tolls and to other data. Results of calculation of T_{tp} and D_{tp} for several available data sets are given in Table 1. It can be seen that the characteristic D_{tp} of victims of natural disasters tends to decrease with time, which can probably be attributed to the global improvement of the technological infrastructures (better quality of civil construction, better hazard mitigation policies etc.).

Table 1 - *Characteristic values T_{tp} and D_{tp} for earthquakes and floods for different world regions and time intervals.*

Disaster type	*Region, country*	*Recurrence time T_{tp}, years*	*Characteristic event, D_{tp}, number of fatalities*	*Maximum event, number of fatalities*	*Data source, data base*
Earthquakes	Developed countries				Significant earthquakes, NOAA, https://www.ngdc.noaa.gov/nndc/struts/form
	1900-1959	33	95000	110000	
	1960-1999	30	24000	17000	
	Developing countries				
	1900-1959	40	270000	200000	
	1960-1999	60	260000	240000	
Floods	North America and European Union				Em-dat, The International disaster database, https://www.emdat.be
	1950-1980	15	1500	650	
	1980-2005	10	500	200	
	SE and S Asia 1984-2006	20	10000	6000	

3. Description of the bend down of heavy tail distribution – detailed examination

A detailed statistical description of heavy tail distribution~~s~~ can be obtained on the basis of the theory of extreme values. This approach needs, however, substantially larger data samples than the simple method discussed above [Pisarenko, Rodkin, 2010; Pisarenko et al., 2010; Pisarenko, Rodkin, 2014; 2015; 2017 et al.].

There are two limit distributions in the extreme value theory: the Generalized Pareto Distribution (GPD) and the General Extreme Value distribution (GEV). Each of these distributions depends on three parameters and they are closely interconnected (see for details [Pisarenko and Rodkin 2010]). We use below the Generalized Pareto Distribution (GPD). The GPD appears as the limit distribution of scaled excesses over sufficiently high threshold values.

The GPD distribution has three parameters: the form parameter ξ, $-\infty < \xi < +\infty$; the threshold parameter h, $-\infty < h < +\infty$; and the scale parameter s, $0 < s < +\infty$.

The GPD distribution function has the form:

$$GPD_h(x\,|\,\xi, s) = 1 - [1 + (\xi/s)\cdot(x - h)]^{-1/\xi}, \qquad \xi \neq 0;$$
$$GPD_h(x\,|\,0, s) = 1 - \exp(-(x - h)/s), \qquad \xi = 0; \tag{11}$$

There is a close connection between GPD and GEV distribution laws. A Poisson flow of events obeying a GPD law is distributed in accordance with the GEV distribution law with the same form parameter ξ. There are simple formulas connecting other parameters of GEV and GPD (see [Embrechts et al. 1997; Pisarenko & Rodkin, 2010]).

Using GPD distribution as limit distribution for a particular data sample involves the estimation of three parameters (11). After this estimation is done, the calculation of any statistical characteristic for the maximum event in any future time interval is a routine procedure. However, the practical use of GPD is often limited by a deficit of available data: our experience shows that a reliable estimation typically requires at least 30 strong events in the uppermost range.

Table 2 presents several estimates of GPD-parameters for a number of disasters (see [Pisarenko & Rodkin, 2014] for details). The value ($-1/\xi - 1$) characterizes for negative ξ the rate of decay of the density to zero in vicinity of the upper limit bound M_{max} :

$$f(x) \sim 1/(M_{max} - x)^{-1/\xi - 1}, \quad x \to M_{max}\,; \quad -1 < \xi < 0.$$

The faster decay rates of the tail to the zero value (Table 2) are observed for the economic losses produced by floods and hurricanes, whereas the corresponding fatality and the injured/affected distributions have, as a rule, smaller $|\xi|$-values, which corresponds to a slower decay of the tail.

The maximum M_{max} of the GPD distribution with negative form parameter ξ equals

$$M_{max} = h - s/\xi, \quad \xi < 0. \tag{12}$$

Thus, the lesser $|\xi|$ the larger M_{max} . Factually it means that in the case of small $|\xi|$ values, the M_{max} is highly unstable and its estimate is not robust. Instead of using unstable M_{max}, we introduced a more stable estimate that characterizes the uppermost range of the distribution, namely, the quantile of the GPD-distribution. The quantile $Q(q)$ of probability level q for the distribution function $F(x)$ is defined by the relation:

$$F(Q) = q. \tag{13}$$

Thus, the quantile Q_q is in fact the inverse function with respect to the distribution function $F(x)$. The continuous distribution function and the quantile function are uniquely related. The distribution function is an integral characteristic of random value (in contrast to the local characteristic – the probability density). So, we can consider the quantile as an integral characteristic of the distribution's tail. That is why the quantile gives more stable and robust characterization of the tail than the point estimation M_{max} eq.(12), see discussion [Pisarenko & Rodkin, 2010; 2014]. Besides, one may interpret the quantile Q_q as an upper confidence bound of level q for corresponding random value x:

$$Pr\{ x < Q_q \} = q. \tag{14}$$

For GPD-distribution the quantile Q_q has the form:

$$Q_q = h + (s/\xi)\cdot[(1-q)^{-\xi} - 1]. \tag{15}$$

We calculated such quantiles $Q_q(T)$ for maximum event size in future time interval T for a number of disasters [Pisarenko and Rodkin, 2014]. Table 2 presents $Q_q(T)$ with confidence level q=0.95 and time interval 10 years $Q_{0.95}(10)$. The intensity of the seismic flow (number of events per unit time) is designated as λ.

Looking at these estimates, one may conclude that economic losses are strongly influenced by a rapid global development of the technological infrastructure and by the population growth. For that reason, a reliable forecast of such characteristics over long time spans is quite problematic. The quantiles are more robust with respect to such uncertainties.

The last two rows in Table 2 summarize the results of the analysis of annualized data. The aggregation of event sizes over one-year intervals represents in essence a linear filtration (smoothing) of the corresponding time series. That is why the tails of annualized distributions are as a rule less heavy compared to the tails of original distributions. This fact can explain the trend for higher absolute values of the form parameter $|\ \xi\ |$ of annualized distributions in Table 2, compared to the corresponding form parameters for individual events.

Table 2 - Characteristics of disasters and form parameter of fitted GPD-law.

	Lowe threshold m_0, Sample size n, Intensity λ, (1/year)	*Form parameter ξ*	*Maximum observed value*	*Quantile $Q_{0.95}$ (10)*
Seismic moment Mw,CMT catalog, 1976-2012	m_0=6.8 n=324 λ=8.8	-0.16±0.08	9.1	9.1
Earthquake fatalities, Japan, 1900-2011	m_0=3 persons n=44 λ=0.339	-0.26±0.11	142 807 persons	58 thousand persons
Earthquake injured, Japan, 1900-2011	m_0=3 n=99 λ=0.884	-0.37±0.06	103733 persons	75 thousand persons
USA, fatalities from floods, 1995-2011	m_0=3 n=41 λ=1.11	-10^{-9}	35 persons	53 persons
Affected in floods, USA, 1995-2011	m_0=500 n=52 λ=3.06	-0.18±0.11	11 million persons	17 million persons
USA, fatalities from tornadoes, 1953-2012	m_0=20 n=53 λ=0.88	-10^{-9}	1200 persons	1500 persons
Annual economic losses from floods in USA, 1940-2011	m_0=2.5 n=48	-0.35±0.09	51.3, 10^9 $	46, 10^9 $
Annual economic losses from hurricanes in USA, 1940-2011	m_0=32 n=64	-0.64±0.05	141, 10^9 $	123, 10^9 $

4. A two-branch model for distribution of earthquake magnitudes

As mentioned above, the practical use of GPD approach is limited by a deficit of available data needed for reliable estimation of unknown parameters. The two-branch model that we introduce below, allows us to partially lift this limitation. The magnitude distribution of moderate size earthquakes is well known to obey the normalized Gutenberg-Richter (G-R) distribution law:

$$F(m) = 1 - exp[-b\cdot(m - m_0)]; \qquad m_0 \leq m$$

In terms of seismic moment values M_0

$$lg(M_0) = 1.5m + 16.1 \quad \{dine\cdot cm\}$$

This law represents a power-law distribution:

$$Pr\{M_0 < z\} = 1 - C/z^{2b/3\ln(10)}\ ;\ C = const. \quad (16)$$

The exponent in (16) is typically less than unity; therefore, this is a distribution with a heavy tail. We discussed above the physical inconsistence of infinite models with $\beta \leq 1$. The family of GPD-distributions includes infinite distributions with heavy tails when $\xi \geq 0$. But for $\xi < 0$ the GPD-distribution is finite. We propose a model with a distribution that coincides with the Gutenberg-Richter model in the lower and intermediate range, and follows the GPD-distribution with $\xi < 0$ in the large event range [Pisarenko and Rodkin 2020].

These two laws are smoothly attached to each other at some point h, so that the overall distribution function $F(m)$ of the two-branch model is:

$$F(m)=\begin{cases} C_1\{1 - \exp[-b\cdot(m - m_0)]\}; & m_0 \leq m \leq h\ ; \\ C_1\{1-\exp[-b\cdot(h- m_0)]\}+C_2\{1-[1 + (\xi/s)\cdot(m - h)]^{-1/\xi}\};\ & h \leq m \leq M_{max}\ ,\ \xi < 0. \end{cases} \quad (17)$$

In (17) the first branch corresponds to the G-R law, and the second one is the GPD law with a negative form parameter $\xi < 0$. Here we a priori consider only negative values $\xi < 0$, which corresponds to the finite distribution of magnitudes, in contrast to models sometimes used for the magnitude distribution. One can note that most of the estimates of ξ from real data sets turned to be negative (see Table 2 and [Pisarenko, Rodkin, 2010; 2014]).

Model (17) contains 5 unknown parameters. The threshold h separates 2 branches of the model; b, m_0, s, ξ are the model parameters; C_1, C_2 are constants that depend on the above parameters and should ensure the normalization of the distribution function $F(m)$ and its continuity:

$$C_1 = 1/\{1 + bs\cdot\exp[-b(h-m_0)] - \exp[-b(h-m_0)]\},$$
$$C_2= 1 - C_1\{1-\exp[-b(h-m_0)]\}. \quad (18)$$

Moreover, we impose that the distribution density function $f(m) = F'(m)$ be continuous at the branches junction point $m=h$. From this condition we get:

$$s = (1+\xi)/b. \quad (19)$$

Finally, we obtain the following two-branch model:

$$F(m) = \begin{cases} C_1\{1 - exp[-b\cdot(m - m_0)]\}; & m_0 \le m \le h; \\ C_3 + C_2\{1 - [1 + \frac{b\xi}{1+\xi}(m - h)]^{-1/\xi}\}; & h \le m \le h - \frac{b\xi}{1+\xi}, \quad -1 < \xi < 0; \end{cases} \quad (20)$$

$$C_1 = 1/(1+\xi\, exp[-b\cdot(h - m_0)]);$$
$$C_2 = (1 + \xi)\, exp[-b\cdot(h - m_0)]\,/(1+\xi\, exp[-b\cdot(h - m_0)]);$$
$$C_3 = \{1 - exp[-b\cdot(h - m_0)]\}/(1+\xi\, exp[-b\cdot(h - m_0)]).$$

The meaning of the parameter m_0 is straightforward: it is the lower threshold of the event sizes that satisfy the G-R law. Thus, we are left with three unknown parameters h, b, ξ that need to be estimated from data. We estimate these parameters by the maximum likelihood method. The number of unknown parameters (three) is the same as in the case of the GEV or GPD distributions discussed above, but now an additional information about moderate size earthquakes is included in the statistical estimation.

This approach was applied to the data on seismicity of Japan and of the Kuril Islands. We would like to emphasize that the spatial resolution of estimates obtained by using the two-branch model can be higher than that obtained by traditional models, since it provides the parameter estimation based on samples of lesser size.

We illustrate the application of the two-branch model on the following example, where the quantile $Q_q(T)$ of maximum earthquake during the future T years with confidence level q is taken as the main characteristic of the seismic hazard. The quantile $Q_q(T)$ means that in the future T years, the maximum earthquake in the considered region will not exceed $Q_q(T)$ with the probability q. The quantiles $Q_q(T)$ for $T = 50$ years, $q = 0.90$ are presented on Fig. 4.

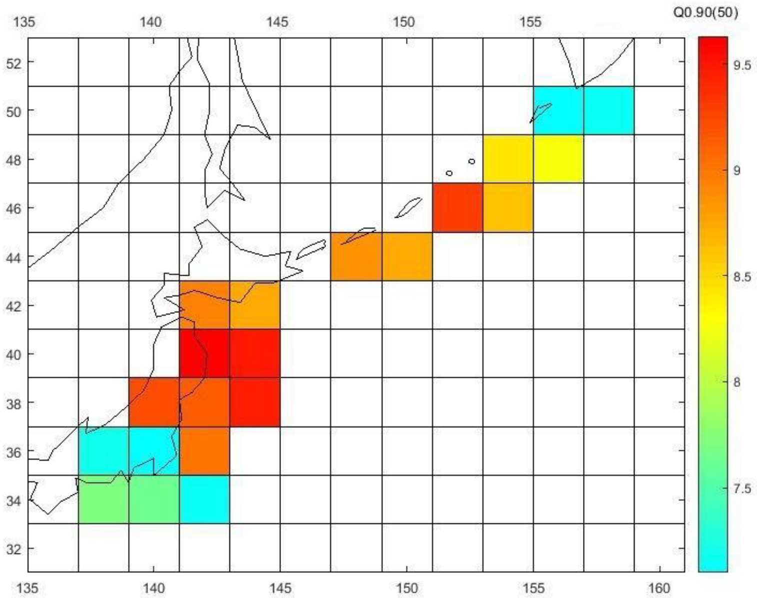

Figure 4 - *The spatial distribution of quantile $Q_{0.9}(50)$ for Japan and the Kuril islands.*

5. Discussion

The power-law heavy tailed distributions have proven to be adequate model for values of damage (such as fatalities) produced by natural disasters, as well as for a number of other characteristics of natural catastrophes. That is true at least for relatively limited time intervals covered by existing catalogs of events. Energy of earthquakes and seismic moments also demonstrate distributions of such kind. The statistical methods described above provide a more or less detailed analysis of the heavy-tailed distributions depending on the amount of available observations. As a first step, the very fact of applicability of the power law distribution can be verified, and the associated exponent β can be estimated. The requirements to the data volume are minimal at this stage. If the exponent for the model power-law distribution is $\beta < 1$ (which occurs quite often in practice), the corresponding distribution has an infinite mathematical expectation. In this case, the standard methods of statistical analysis based on sample mean and sample standard deviation are not applicable, and one can obtained erroneous estimates by using these statistics.

Naturally, all real losses and earthquake energy values are finite. Therefore, the distribution law should deviate from the Pareto law with $\beta<1$ in the range of rare strong events. A more detailed analysis can be performed with the help of models using the concepts of the transition point T_{tp} and the characteristic event size D_{tp} (damage or magnitude). The transition point T_{tp} indicates the time moment when the nonlinear damage growth caused by the effects of the heavy tail becomes linear with respect to time. The characteristic size D_{tp} can be determined through the recurrence time T_{tp}. The data needed for performing such an analysis may include only several events in the extreme range.

Estimates of T_{tp} and D_{tp} for earthquakes and floods related fatalities from event catalogs covering the period 1900-2016 show a decreasing trend for their values with time. One can therefore conclude that the characteristic number of victims resulting from a natural disasters tends to decrease with time, contrary to a wide spread point of view. Similar conclusion can also be made for earthquake death tolls. Observations testify for a certain general decrease with time of fatalities associated with a characteristic natural disaster. These conclusions are in contradiction with some inferences published earlier [Osipov, 1999; 2002].

More detailed analysis of damage distribution can be performed on the basis of the theory of extreme values. However, such an analysis requires a significantly bigger amount of data: practical estimates show that the registered large events should count at least in dozens.

The application of these methods in practice has demonstrated their effectiveness. It can be noted, that in the majority of cases we obtained negative values of the GPD form parameter ξ, which corresponds to a finite distribution with the upper bound M_{max}, see eq.(12):

$$M_{max} = h - s/\xi\,, \qquad \xi < 0,\ \ s > 0.$$

Such a result was to be expected: since any real values of fatalities, losses, and energy of disasters are finite, the corresponding distribution law must be finite too. Quite often the obtained estimates of ξ are very close to zero, which gives rise to an instability of the estimation of M_{max}. That explains the uncertainties in the estimation of the maximum regional earthquake magnitude, which in turn leads to the necessity of regular revisions of the seismic zoning maps. The same reason is behind the non-robustness of estimates of the maximum damage values (number of victims) from natural and man-made disasters.

In contrast to non-robust estimates of M_{max}, the quantiles $Q_q(T)$ – the sizes of maximum event for T future years expected with probability q - are robust. The associated damage characteristics T_{tp} and D_{tp} are robust as well.

Practical application of the theory of extreme values encounters limitations due to the lack of strong events in the available observations that are needed for the estimation of GPD- and GEV-parameters. Application of GPD- and GEV- approaches shows that one typically needs at least 30 strong events for a reliable estimation of their parameters. We therefore suggested to use the information contained in moderate size events, and to cover the "strong events" and "moderate events" intervals of the distribution by a common recurrence law. To that end, we introduced the two-branch earthquake size distribution model: the Gutenberg-Richter law is used for the moderate size earthquakes, while the strong earthquakes are modelled by the GPD distribution. Using this approach, we constructed the map of the quantiles $Q_q(\tau)$ of strong earthquakes for Japan and Kuril Islands area which has the spatial resolution approaching that of the seismic zoning maps. Taking into account the similarities of the earthquake size distribution with the distribution of fatalities and disaster related losses, we believe that this approach can also be applied to such data.

6. Conclusion

The heavy tailed distributions are commonly used for modelling of losses from natural disasters (in particular, the number of fatalities). The distribution of seismic moments of earthquakes is also frequently modeled by the power law distribution with a heavy tail. We presented in this paper a collection of statistical methods for studying distributions with heavy tails. These methods allow us to describe the empirical data with the help of statistical models whose degree of detail depends on the amount of the available observations. We demonstrated how the exponent of the approximating power law can be adequately estimated in the case of limited data sets. We also showed how to obtain rough estimates of the transition time T_{tp} from a non-linear to a linear mode of growth of cumulative loss, and the estimates of the characteristic damage size D_{tp}. For larger data samples, the methods of the theory of extreme values can be applied.

Acknowledgements.

We would like to express our deepest gratitude to the anonymous Reviewer, whose numerous, valuable remarks and comments have made the presentation of our paper more clear and correct.

The researches were fulfilled within the framework of the state task: topic AAAA-A19-119011490129-0 “Development of seismic data analysis in order to study the source, environment, seismic hazard” and partially supported by the Russian Foundation for Basic Research, grants №20-05-00351, 17-05-00351 and 19-05-00466.

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Increasingly, socio-natural risks and disasters represent the result of an unsustainable interaction between human beings and environment. The current scientific debate has generally agreed on the idea that the impact of natural hazards needs to take into account the social vulnerabilities and exposures to risk of the affected population. The most recent earthquakes have unequivocally shown the complexity of the phenomena and their multi-scale dynamics. Indeed, the territory is the combination of natural, social and cultural environment and only by exploring its anatomy and physiology, it will be possible to manage and protect it in the best way.
This volume collects a quite wider range of national and international case studies, which investigate how socio-natural risks are perceived and communicated and which strategies the different communities are implementing to mitigate the seismic risk. This publication has been possible thanks to a fruitful discussion that some scholars had at the 36th General Assembly of the European Seismological Commission held in Malta from 2 to 7 September 2018.

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